The acoustic transfer function at the surface of a layered poroelastic soil

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A model for the response of a poroelastic layered soil to an incident plane wave developed in a previous paper [Sabatier *et al.*, J. Acoust. Soc. Am. 79, 1345–1352 (1986)] is used to predict the complex sound pressure within the upper poroelastic layer. The predictions both of phase velocity and attenuation of the slow wave associated primarily with propagation in the pore fluid are compared with measurements made with a specially constructed probe microphone. The agreement between theory and experiment is good. The predictions for a layered poroelastic soil model are compared numerically with those of a semi-infinite rigid porous soil model and are found to differ only at frequencies higher than 1000 Hz. Analysis of the sensitivity to the theoretically predicted propagation constants in the poroelastic soil to the assumed value for the bulk rigidity modulus of the soil predicts that over the known range of rigidity moduli for soils it is possible to obtain a switchover between fast and slow propagation modes. This switchover occurs at the lower end of the possible range of values of the shear modulus. It is suggested that probe microphone measurements in air-filled soils offer a way of measuring flow resistivity and of deducing the structural parameters required for application of the Biot–Stoll model to water-saturated sediments.

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INTRODUCTION

For many years researchers have assumed that the ground surface is acoustically locally reacting and have used an empirical relationship for outdoor ground impedances derived by Delany and Bazley.[1] These expressions are based on the assumption that the ground behaves acoustically as a modified fluid, and they predict the normal surface impedance and the propagation constant from the bulk flow resistivity only. In other instances the ground has been modeled as a semi-infinite rigid porous medium,[2] as a variable porosity rigid-porous medium,[3,4] or as a hardback rigid porous layer.[5,6] These models require additional parameters for the determination of the characteristic impedance and the propagation constant in the porous soil which are needed, in turn, to determine the surface impedance. In a previous publication,[7] the surface of the ground was modeled as that of a poroelastic soil layer of known thickness overlying a semi-infinite nonporous elastic substrate. The poroelastic layer permits two compressional wave types called fast and slow, respectively. In this paper, the propagation constant of the slow wave determined from the poroelastic semi-infinite and layered models are compared to those of the semi-infinite rigid frame model. A probe microphone suitable for measuring the attenuation and phase speed of the acoustic slow wave in the porous soil has been designed. Measurements made at several outdoor sites have been compared to theoretical predictions. Some of these comparisons have been published previously.[8] The results of the attenuation measurements using the probe indicate that, in many cases, the sand and soils investigated are not acoustically homogeneous. Probe measurements also reveal extremely slow speeds of propagation of the acoustic wave in porous soil, which supports the common assumption of local reaction. This property means that there is negligible difference between the subsurface acoustic fields due to the incidence of either spherical or plane waves as has been predicted elsewhere.[9]

I. ACOUSTIC TRANSFER FUNCTION

The acoustic transfer function of the ground surface may be defined as the ratio of the acoustic pressure at some specified depth below the surface to that above the surface. Using the poroelastic layer model described previously,[7] the acoustic pressure above the surface results from the incident and reflected waves and may be expressed as

$$P_{\text{above}} = iK_f l_i (B_i + B_r)\cos\theta_i\,, \tag{1}$$

where B_i and B_r are the amplitudes of the incident and reflected waves, K_f is the bulk modulus of the air, l_i is the propagation constant for the incident wave, θ_i is the angle of incidence, and $i=\sqrt{-1}$. The pore fluid pressure below the boundary results from four waves in the poroelastic layer. These are transmitted and reflected fast and slow waves. Using the notation of Ref. 7, the pressure below the boundary at depth $z=d$ is

$$\begin{aligned}P_{\text{below}} &= iA_1 l_1 (C - m_1 M)\exp(il_1\cos\theta_1 d) + iA_2 l_2 (C - m_2 M)\exp(il_2\cos\theta_2 d)\\ &\quad + iA_1' l_2 (C - m_1 M)\exp(-il_2\cos\theta_1 d) + iA_2' l_2 (C - m_2 M)\exp(-il_2\cos\theta_2 d)\,.\end{aligned} \tag{2}$$

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In this notation, the two compressional wave propagation constants are denoted by l_1 and l_2. The elastic constants of the composite medium are denoted by H, C, and M. The plane wave solution for the boundary conditions, including shear waves, requires nine plane waves. However, only four of the waves contribute to the pore fluid pressure. These are the transmitted and reflected fast and slow waves in the layer itself for which the amplitudes are denoted by A_1, A_2, A'_1, and A'_2, where the prime indicates reflected waves and the θ_i's are the associated angles of refraction and reflection. In Stoll's notation the m_i's are the ratios of the relative fluid displacements to the frame displacements for the fast and slow waves.

In this notation the acoustic transfer function between the surface and depth d is a complex quantity expressed as

$$T_a(d) = P_{\text{below}}(d)/P_{\text{above}}(0) . \tag{3}$$

By calculating the acoustic transfer function for two different depths below the surface, both the pore fluid attenuation and phase speed can be determined for the poroelastic layer. For depths of z_1 and z_2 $(z_2 > z_1)$, the frequency-dependent attenuation coefficient may be expressed as

$$\alpha(\omega) = [\ln|T_a(z_2)| - \ln|T_a(z_1)|] \times 20 \log_{10} e/(z_2 - z_1) , \tag{4}$$

where ω is the angular frequency. The phase speed was also calculated from the acoustic transfer function at two depths below the ground surface and may be expressed as

$$c(\omega) = \{\tan^{-1}[\text{Im}(T_a)/\text{Re}(T_a)]\}^{-1}(z_2 - z_1)\omega . \tag{5}$$

For the semi-infinite poroelastic treatment, the acoustic pressure below the surface may be expressed more simply, since there are no waves returning from a lower boundary. The pressure at a depth d below the surface is expressed easily from Eq. (2) as

$$P_{\text{below}} = iA_1 l_1 (C - m_1 M) \exp(il_1 \cos\theta_1 d) + iA_2 l_2 (C - m_2 M) \exp(il_2 \cos\theta_2 d) . \tag{6}$$

The corresponding transfer function, attenuation, and phase speed are given by Eqs. (3)–(5) after noting that the pressure above the surface is given by Eq. (1). The complex transfer function for plane waves incident at angle θ_i on a semi-infinite rigid porous interface may be written simply as

$$T_a(d) = \exp[il_i d(n^2 - \sin^2\theta_i)^{1/2}] , \tag{7}$$

and $n = k_b/l_i$ is the complex refractive index of the rigid porous medium, k_b being the complex wavenumber within the medium. The only permitted wave motion within the rigid porous medium represents a limiting case of the acoustic slow wave within a poroelastic medium of the same flow resistivity, porosity, and pore structure. Analytical expressions for k_b may be found elsewhere.[2] Figure 1 shows comparison of the magnitude of the complex transfer function as predicted by each f the models described above. The parameter values used correspond to those tabulated in Ref. 7 for sandy soil.

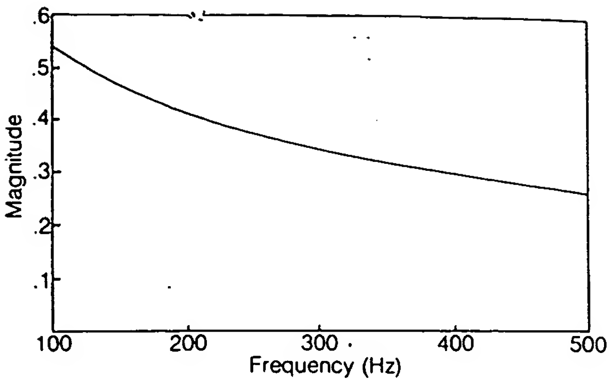

FIG. 1. Magnitude of the complex transfer function predicted for a layered poroelastic soil (layer thickness 2 m), for a semi-infinite poroelastic soil, or for a semi-infinite rigid porous soil. Results to the accuracy of this graph are equal.

II. MEASUREMENTS OF THE ACOUSTIC WAVE IN THE EARTH

The measurements of sound pressure below the surface of the earth were made with a specially designed probe shown in Fig. 2. It consisted of an AKG microphone and preamplifier housed in a cylindrical brass tube. The inner cylinders shown were used to increase the mass of the probe and to seal any possible leaks. The nose cone contains small holes to allow the element of the microphone to sense the acoustic field. A rubber membrane was stretched over the microphone element to protect it from any particles or moisture that might penetrate the nose cone when the probe is inserted into a predrilled hole. A comparison of the probe microphone response to a standard AKG microphone is shown in Fig. 3. The frequency response of the probe is essentially flat up to 3 kHz where it begins to roll off to a new, less sensitive, plateau near 7 kHz. The frequency response of the probe is essentially unchanged when the probe is pointed away from the sound source, the normal geometry for measurements below the earth's surface.

Before the probe was inserted into the ground, a drill was used to bore a hole 1 in. in diameter to the desired depth. The probe was then forced into the hole and highly viscous

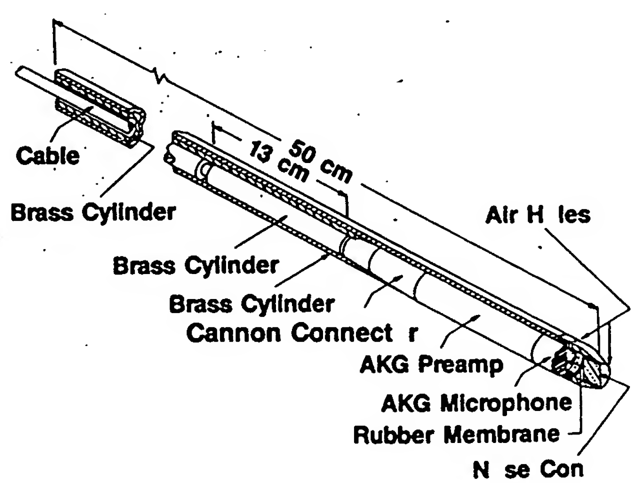

FIG. 2. The design of a probe microphone for measurements beneath the soil surface.

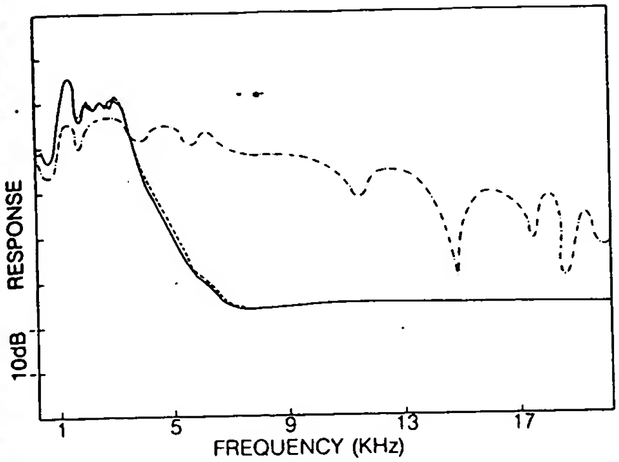

FIG. 3. Comparison of free-field response of probe and standard AKG microphones, AKG microphone (- - -), probe facing source (—), probe facing away from source (- - -).

oil treatment was poured around the probe near ground level to further seal the probe entry. This procedure worked well, giving reproducible results for sandy soil and loam.

For clays, the soil in the vicinity of the probe would quite often crack, providing air paths not previously available. This cracking gave results which differed from one attempt to the next. It was generally assumed in such conditions that the measurement giving the smallest sound pressure at the probe was best. Occasionally the holes in the tip of the probe became filled with dirt when the probe was inserted. This occurrence could only be detected when the probe was removed and examined, so several data runs had to be rejected after the fact. In sand it was not necessary to drill holes as the probe could be pushed easily into the ground. Furthermore, greater reliance could be placed on individual measurements with regard to the sound path.

Measurements for the attenuation and phase speed in the frequency range 50–2500 Hz have been made at several sites. Initially, a hemispherical dish (radius 0.5 m) was formed in the ground of mean flow resistivity 500 000 mks rayls/m and filled with sand which had a measured flow resistivity[10] of 85 000 mks rayls/m. Among the other sites, results are presented here of measurements made at a sand quarry and a large, flat area of dredged sand which had been weathered over several years and which supported sparse growths of lichen and vegetation. Table I lists the physical properties measured at various locations on this site. The acoustic attenuation at all sites was determined in the following manner.

A reference acoustic spectrum was recorded using swept tones transmitted through a small speaker, received by a probe microphone at the sand surface, and processed through a scanning filter. The probe was pushed then to a depth of d meters and the swept tone repeated. The attenuation in dB per meter was calculated from the difference in the two spectra per unit length. The measurements were repeated with the probe at several depths up to 20 cm as shown in Fig. 4.

TABLE I. The physical properties measured at various locations on the dredged sand site.

	Porosity (dry)	Mean flow resistivity ($N\,s\,m^{-4}$)
Location 1	0.352	116 000
Location 2	0.314	70 900
Location 3	0.281	25 000
Location 4	0.284	41 500

Above the dredged sand, white noise was broadcast and the acoustic transfer function was measured by recording simultaneously the response of a reference AKG microphone at the surface and probe microphone at a known depth below the surface. Using a spectrum analyzer, the difference in the phase was determined for the surface and a series of probe depths. This difference is directly related to the phase speed of the acoustic wave between two depths. The phase speed is calculated using the expression

$$v(\omega) = \omega \Delta x / \Delta \phi \,, \tag{8}$$

where Δx is the probe separation for the two depths and $\Delta\phi$ is the phase change in radians for the two depths.

Another technique was used to measure the phase speed of the acoustic wave in the hemispherical sand dish. A dual-channel digital oscilloscope was used to measure the delay between the reference AKG and the probe when a pure tone was broadcast. The probe was pushed further into the ground and the delay measured again. From the difference in the delay times and the change in the probe depth, the phase speed was obtained directly as the ratio of the distance and time measurements. Figure 5(a) shows the phase speeds measured in the hemispherical sand dish.

Typically, in homogeneous sands like that contained by the hemispherical dish, the attenuation with depth was uniform. However, probe measurements in weathered soils, sand, and clays reveal a variation of rate of attenuation with depth that appeared not to be the only result of air leaks. The attenuation per centimeter increased with depth below many surfaces as shown in Fig. 4. But several measurements at one location showed that the attenuation over the first 2-cm depths was greater than that of any other equal spacing below those 2 cm. In these instances, a load-bearing sand crust of approximately 1 cm thick had formed. Below this crust the sand was quite loose.

To enable comparison of predictions with these measurements, the various soils were sampled and mean flow resistivities and air porosities were determined. The flow resistivities were measured with a modified Leonard's apparatus[10] and the air porosities were obtained by weighing a known volume on site, drying, and weighing again.

For each of the three sites investigated the measured and theoretically predicted attenuations are shown in Figs. 4, 5(b), and 6. The predicted values were obtained from the rigid porous theory. Figures 5(a) and 7 show the comparison of measured and predicted phase speeds of the induced acoustic wave at two sites.

III. DISCUSSION AND CONCLUSIONS

In Fig. 1, the magnitudes of the acoustic transfer functions calculated using the semi-infinite and layered poroelastic models and the rigid frame model differ by less than 1%.

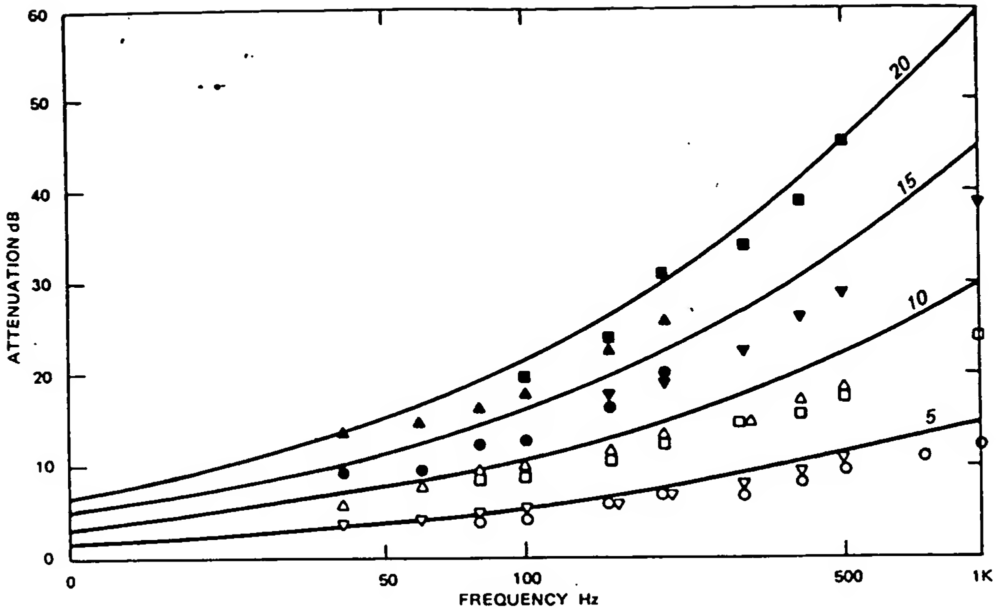

FIG. 4. Attenuation (dB/cm) versus depth in a sand quarry. Measurements were made with probe at 5 cm (▽,○), 10 cm (□,△), 15 cm (●,▼), 20 cm (■,▲).

The comparison of the acoustic attenuation predicted by the three theoretical models described implies that, for most cases, the more rigorous poroelastic theory is not necessary to predict the acoustic attenuation within outdoor ground surfaces. Moreover, the slow wave speeds, both predicted and measured in porous soils (Figs. 5 and 7), confirm the hypothesis for the explanation of local reaction advanced earlier.

For a realistic range of flow resistivities, the energy associated with the wave propagating primarily in the pores is refracted strongly towards the normal to the surface. The impedance then becomes independent of the angle of incidence and regions of the surface, distant from the point of the surface at or below which the sound field is probed, do not contribute to the reflected or refracted acoustic fields. This is difficult to understand if the wave speed in the earth is taken to be the fast or *p*-wave speed which is of the order of hundreds of meters per second, but follows readily from Snell's law if the wave speed in the pores is only tens of meters per second as has been measured. The locally reacting nature of the ground surface is a natural consequence of the porosity of the upper layer. The high attenuation of the penetrating acoustic wave also explains why the actual seismic profile of many grounds may be ignored when considering sound propagation near their surfaces.

For typical flow resistivities, surface layers with thickness greater than 0.2 m will behave acoustically as though they are infinitely thick. Furthermore, as has been demonstrated in both this paper and the previous one,[7] the bulk moduli of typical soils are sufficiently high that, acoustically, they may be regarded as rigid. These results support the applicability of relatively simple models for the acoustical properties of ground surfaces as have been developed elsewhere.[4] When considering acoustically induced soil particle motion, however, the poroelastic models are essential.

It is noticeable that the measured attenuations per centimeter ver the uppermost 2 cm in dredged sand differ considerably from those predicted using the measured mean flow resistivity and porosity (Fig. 6). On the other hand, adjustment of the flow resistivity, increasing its value to fit the lowest frequency data point, enables tolerable agreement

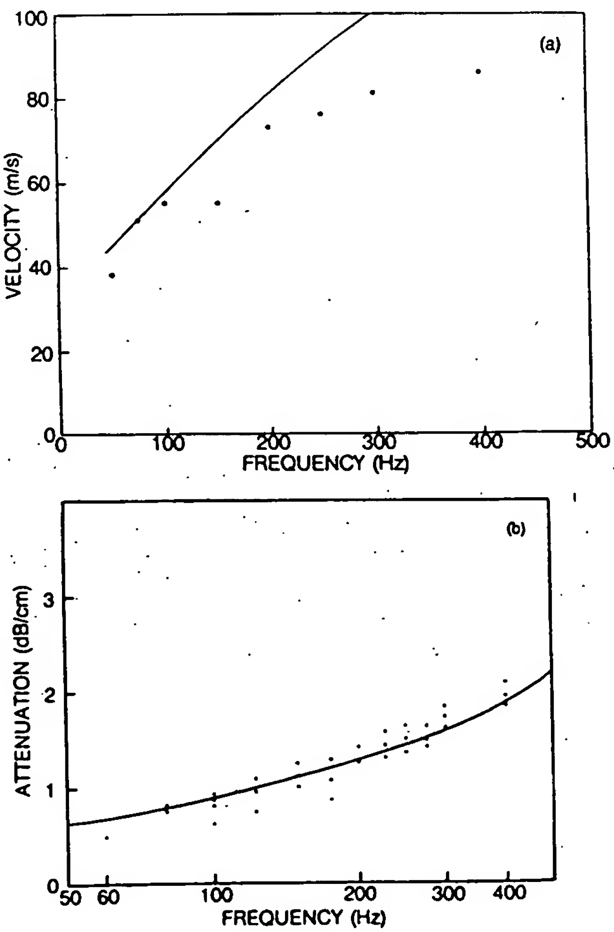

FIG. 5. (a) Measured and predicted phase speed in a hemispherical sand dish. (b) Measured and predicted attenuation in a hemispherical sand dish.

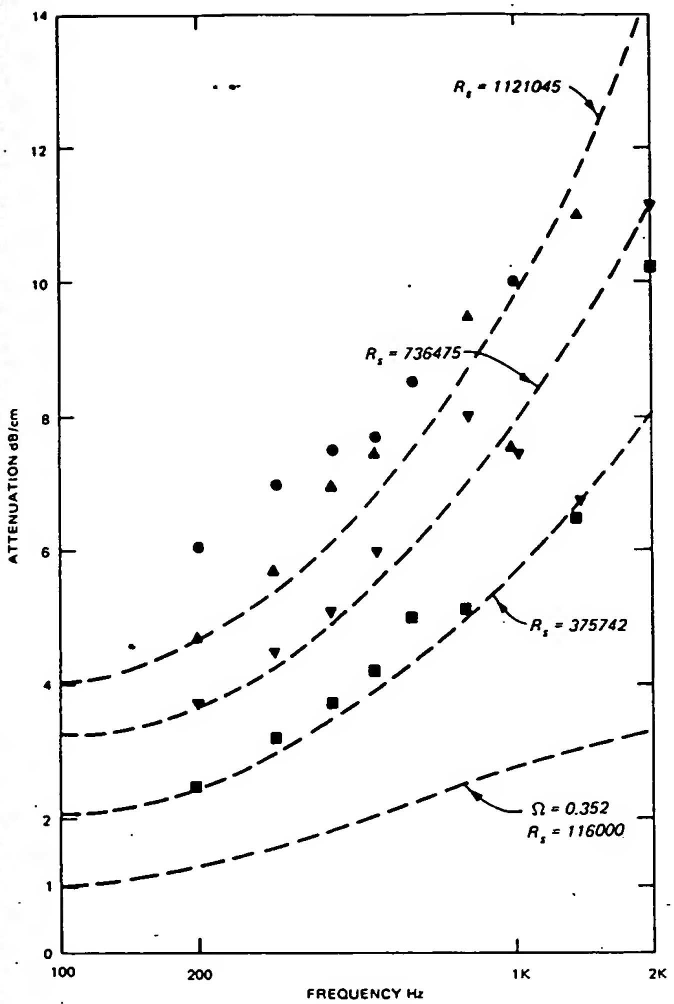

FIG. 6. Measured and predicted attenuation in dredged sand. Theoretical calculations based on Eqs. (4) and (7) and Ref. 2 using the specified values of porosity (Ω) and flow resistivity (R_S). Measurements made with probe at 2 cm for four different locations at the dredged sand site.

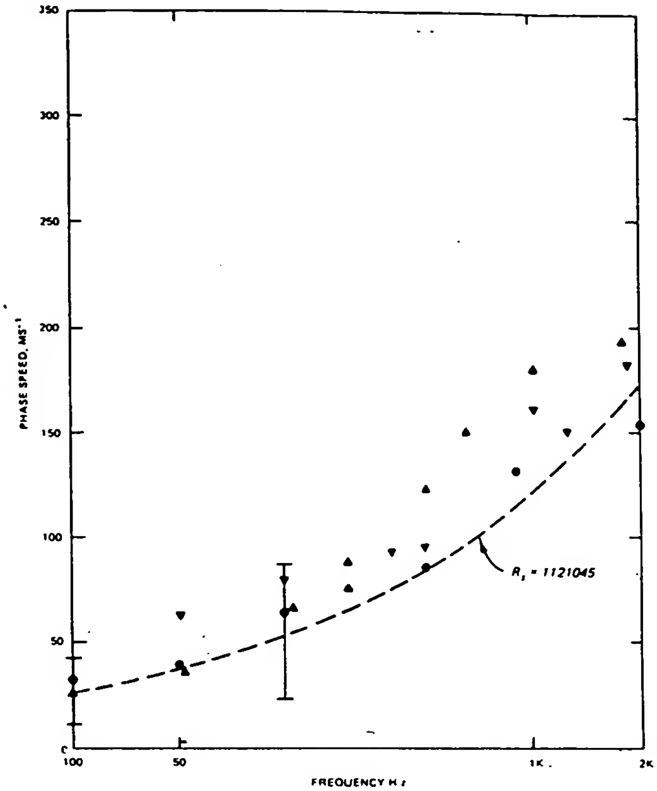

FIG. 7. Measured and predicted phase speed in dredged sand. Measurements were made between 2- and 4-cm depths. Predictions made as in Fig. 6. Error bar is deduced from attenuation measurements made at these depths.

between theory and data over the whole frequency range of measurement. Furthermore, use of these adjusted values of flow resistivity produces theoretical predictions of phase speed that are in good agreement with measurements (Fig. 7). This suggests that the measured flow resistivity is likely to be in error for such sandy soil. This may be a consequence of the invasive technique used, by which a sample is removed, thus loosening its structure and reducing its apparent flow resistivity.

Given the otherwise good agreement that has been shown between measured and predicted attenuations and phase speeds, the possibility arises of using acoustic measurements of the kind reported here as an *in situ* technique for measuring flow resistivity. This would provide useful data both for outdoor sound propagation and for soil science.

It should be noted that the assumption has been made that the acoustic wave in the pores is the slowest of the two compressional waves possible in poroelastic soil. Indeed, for the soils studied here this is the case. However, it is not necessarily always the case. Figure 8 shows the result of reducing the shear modulus while keeping the remaining parameters constant as listed in Table II. A switchover between the two compressional modes is predicted at high frequencies as the shear modulus is reduced. Consequently, it would be possible in a loosely packed soil with high compressibility and low rigidity for the frame wave to be the slowest in some range of frequencies. This is the case, in fact, for many polyurethane foams, as has been observed elsewhere.[11]

A final remark about the acoustic wave in air-filled soils concerns the ease with which it may be measured and pre-

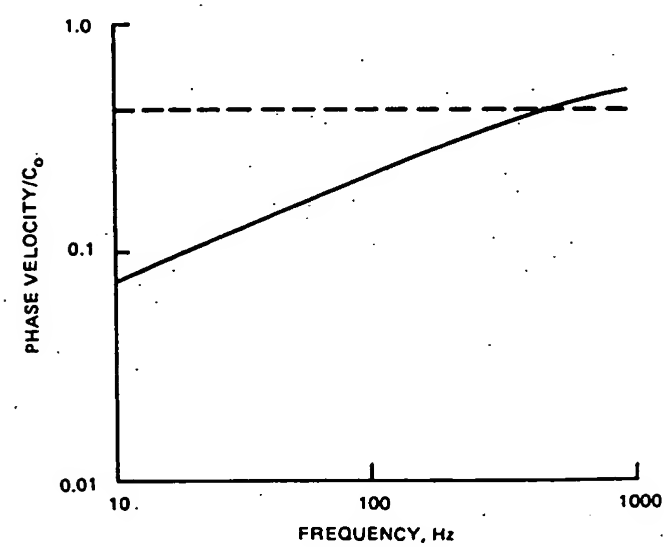

FIG. 8. Effect of reducing shear modulus to 1.38×10^8 dyn/cm² on fast (---) and slow (—) wave speeds in poroelastic soil.

TABLE II. Parameters used for Fig. 8 calculations.

Parameter	Sandy soil
Flow resistivity (N s m^{-4})	78 300
Porosity (dry)	0.4
Frame bulk modulus (N/m^2)	8.27×10^8
Grain shape factor	1
Shape factor ratio	0.75
Grain density (kg/m^3)	2 650

dicted at audio frequencies, in comparison with its counterpart in water-filled sediments whose existence was not verified at ultrasonic frequencies until fairly recently.[12] The fact that the acoustic wave dominates the energy transfer across the boundary of an air-filled porous material, whose frame is relatively rigid, means that measurements of sound reflection and transmission at the surface of fibrous absorbents have demonstrated its existence since the time of Lord Rayleigh.[13] Consequently the existence and properties of the acoustic slow wave in a porous material were known well before their prediction by Biot.[14] Furthermore, values of structural parameters, introduced by the Biot theory, that are independent of the saturating fluid, may be determined by studies of air-filled rather than water-filled media.

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